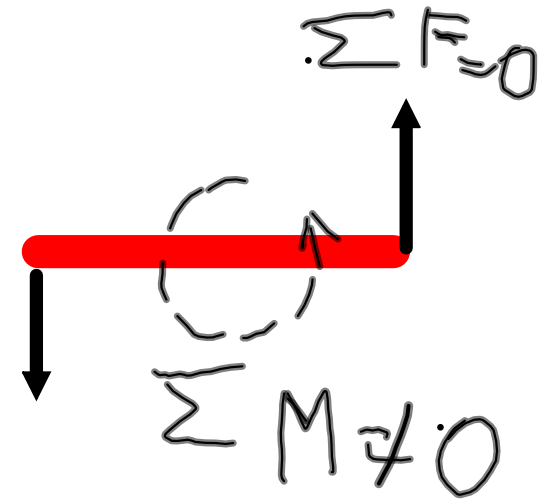


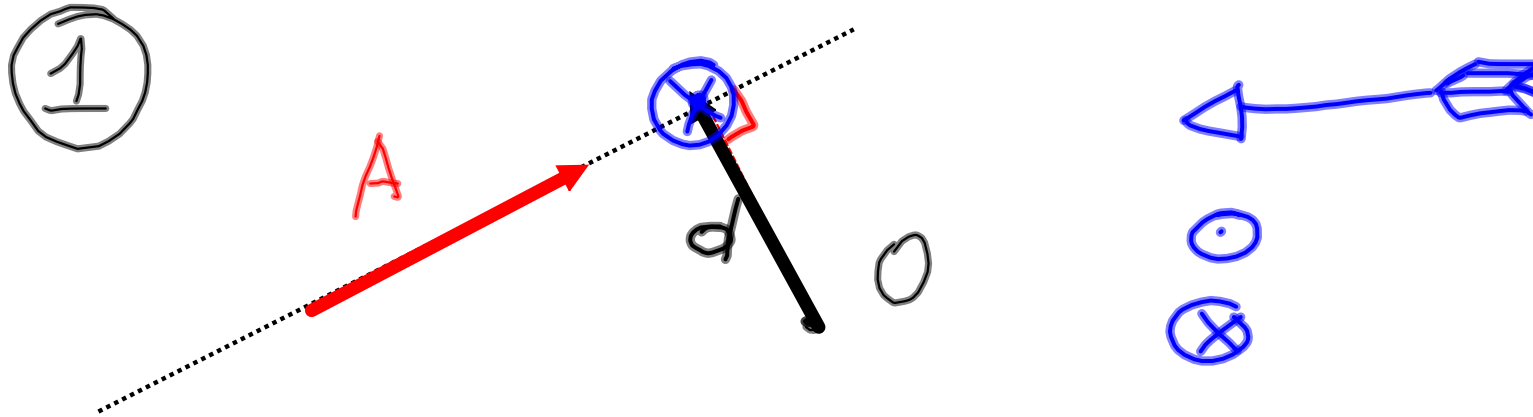
MOMENTO DI UNA FORZA

Movimenti possibili
nello spazio
2D, 3D, ... \Rightarrow $\left\{ \begin{array}{l} \text{TRASLAZIONI} \\ \text{ROTAZIONI} \end{array} \right.$

$\left\{ \begin{array}{l} \text{TRASLAZIONI} \Rightarrow \Sigma F \neq 0 \\ \text{ROTAZIONI} \Rightarrow \Sigma M_P \neq 0 \end{array} \right.$

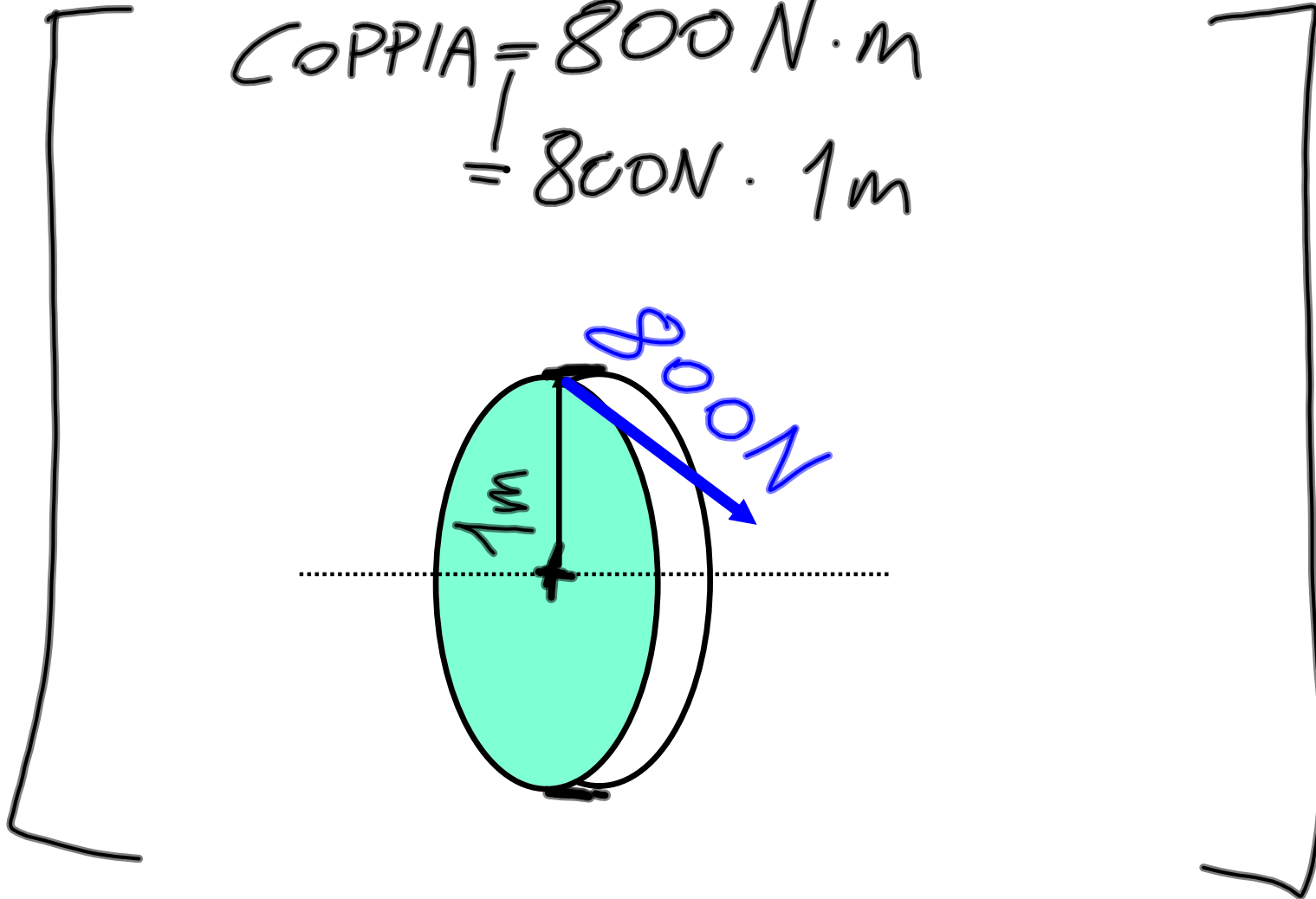


MOMENTO { ① FORZA x BRACCIO
 ② COPPIA/MOMENTO



$$\vec{M}_o = \vec{A} \times \vec{d}$$

$$\begin{aligned} C_{OPPIA} &= 800 \text{ N} \cdot \text{m} \\ &= 800 \text{ N} \cdot 1 \text{ m} \end{aligned}$$



PRODOTTO
SCALARE = OUTPUT : NUMERO x U.d.m.

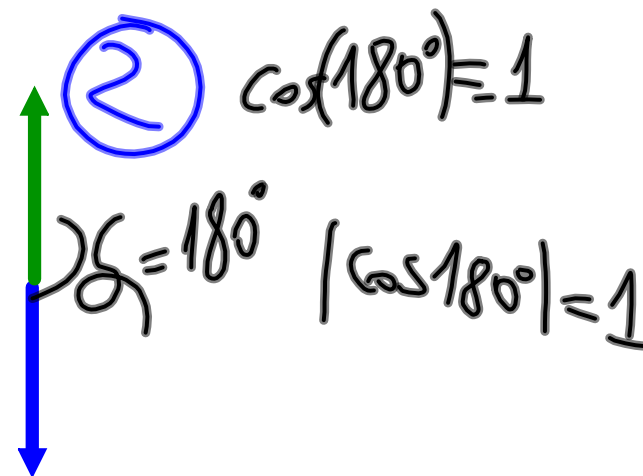
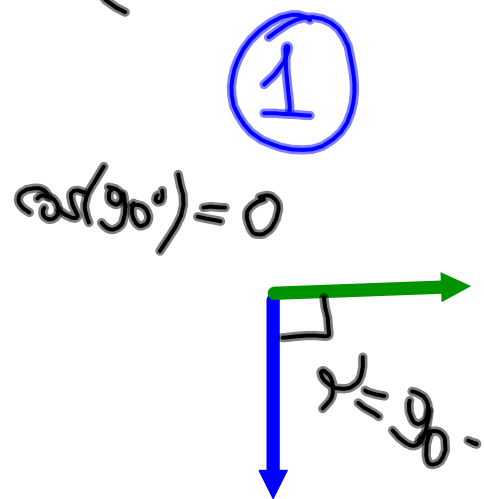
LAVORO = FORZA · SPOSTAMENTO

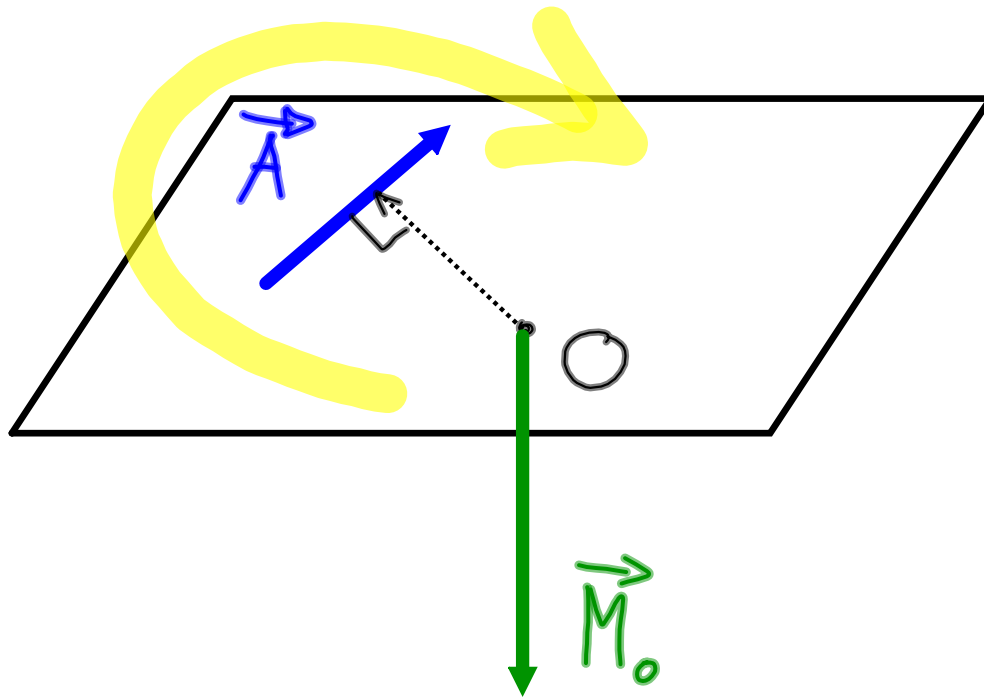
PRODOTTO
VETTORIALE = OUTPUT : VETTORE

$$\text{LAVORO} = \text{FORZA} \cdot \text{SPOSTAMENTO}$$

$$= |F| \cdot |S| \cdot |\cos \alpha|$$

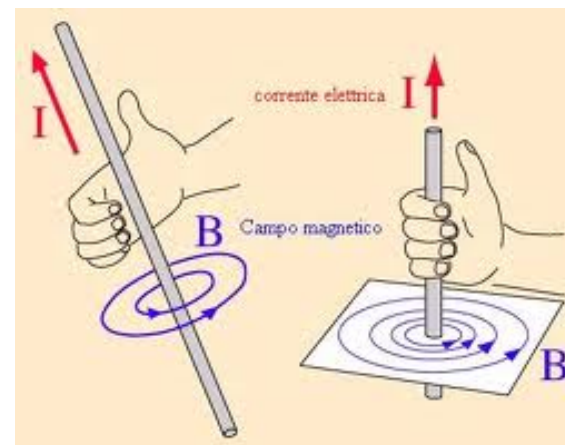
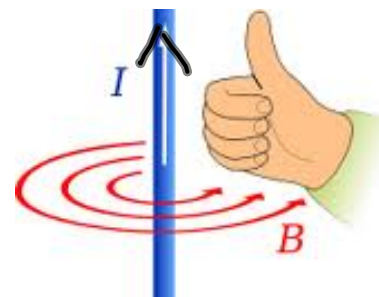
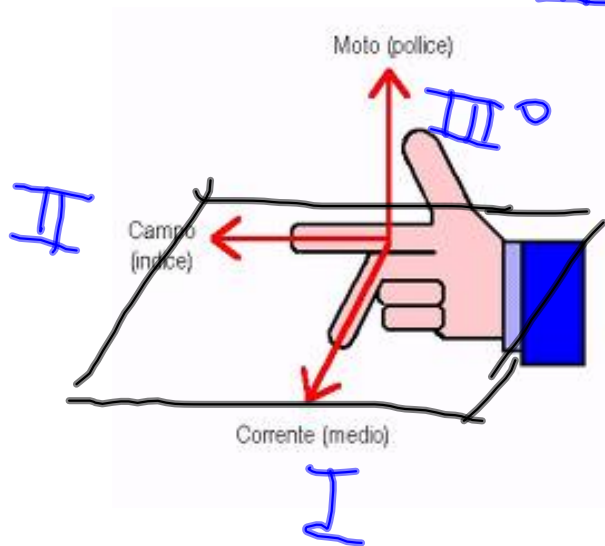
{ P. SCALARE: .
 { P. VETTORIALE: X





$$\vec{M}_o = \vec{A} \times \vec{d}$$

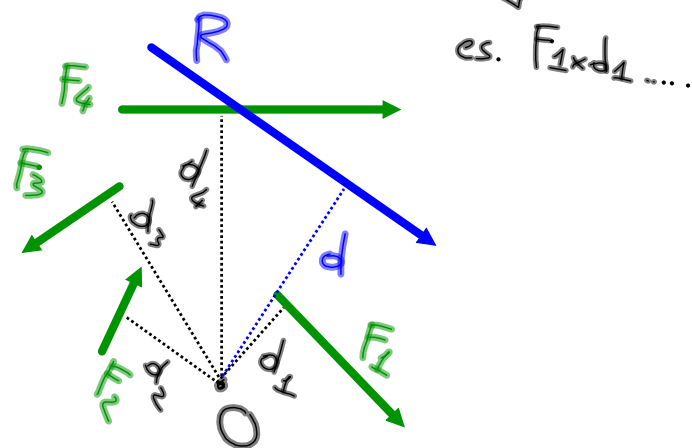
$$\vec{H} = \vec{I} \times \vec{B}$$



Teorema di Varignon

Dato un sistema di forze complanari e scelto un punto nel piano, si può calcolare il momento di ciascuna forza e determinare il momento risultante; ma i singoli momenti e il momento risultante devono soddisfare il teorema di Varignon, per il quale:

"In un sistema di forze complanari il **momento della risultante**, rispetto a un punto "O" qualsiasi nel piano, è uguale alla somma algebrica dei momenti delle singole forze rispetto al piano stesso."



$$F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4 = R \times d$$

$$d = \frac{F_1 \times d_1 + F_2 \times d_2 + F_3 \times d_3 + F_4 \times d_4}{R}$$

ESERCIZIO

Dati i seguenti punti
A,B,C,D,O calcolare la
risultante ed il momento
della risultante rispetto al
punto "O" con riferimento ai
seguenti vettori:

\overline{AB} e \overline{CD}

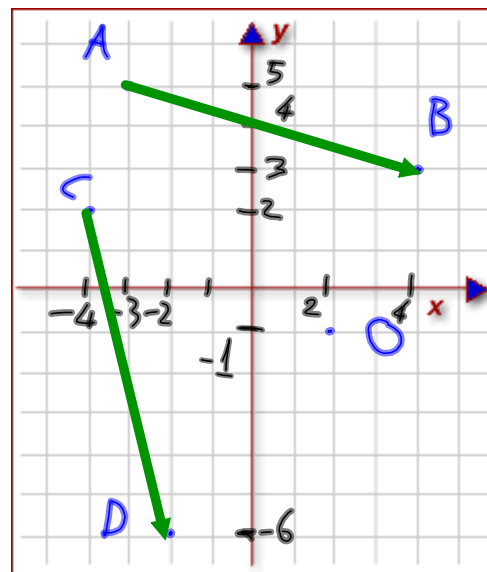
A(-3;5)

B(4;3)

C(-4;2)

D(-2;-6)

O(2;-1)



equazione della retta passante per due punti generici "A" e "B":

$$\frac{x - x_B}{x_A - x_B} = \frac{y - y_B}{y_A - y_B}$$

- A(-3;5)
- B(4;3)
- C(-4;2)
- D(-2;-6)
- O(2;-1)

$$\frac{x-4}{-3-4} = \frac{y-3}{5-3}$$

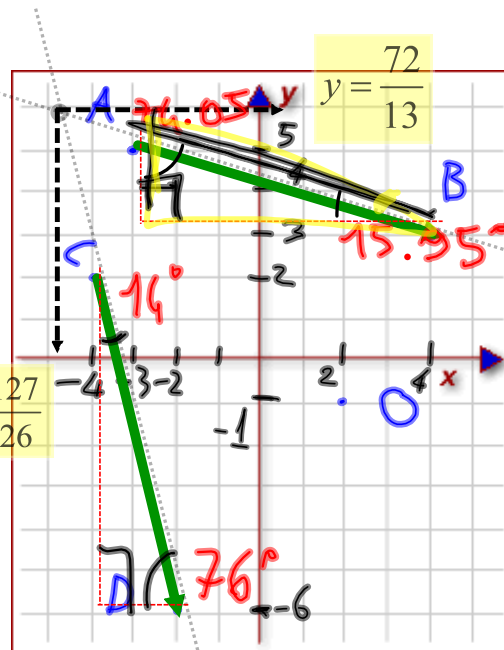
$$\frac{x-4}{-7} = \frac{y-3}{2}$$

$$\frac{-2(x-4) - 7(y-3)}{14} = 0$$

$$-2x + 8 - 7y + 21 = 0$$

$$-2x + 8 + 21 = 7y$$

$$\frac{-2x + 29}{7} = \frac{7y}{7} \rightarrow \frac{-2x + 29}{7} = y$$



$$\frac{\overline{AB}}{\sin \hat{\alpha}} = \frac{\overline{AC}}{\sin \hat{\beta}}$$

equazione della retta passante per due punti "A" e "B":

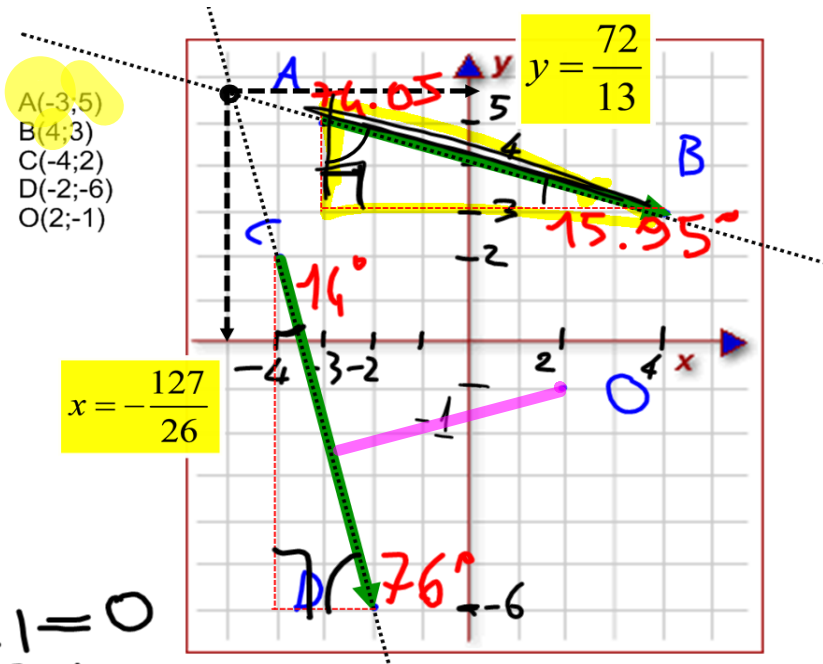
$$\frac{x - x_B}{x_A - x_B} = \frac{y - y_B}{y_A - y_B}$$

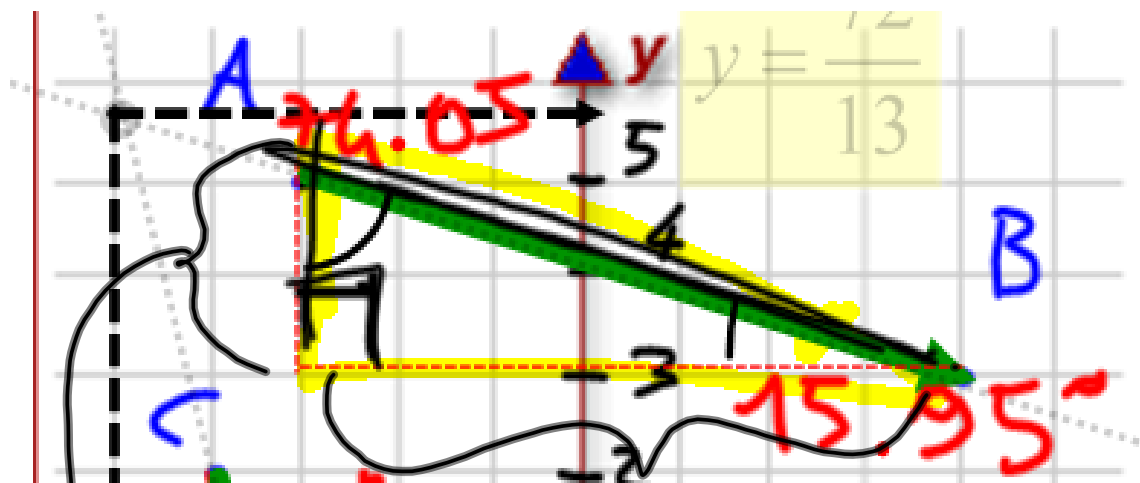
$$\frac{x - 4}{-3 - 4} = \frac{y - 3}{5 - 3}$$

$$\frac{x - 4}{-7} = \frac{y - 3}{2}$$

$$-2(x - 4) - 7(y - 3) = 0$$

$$-2x + 8 - 7y + 21 = 0$$



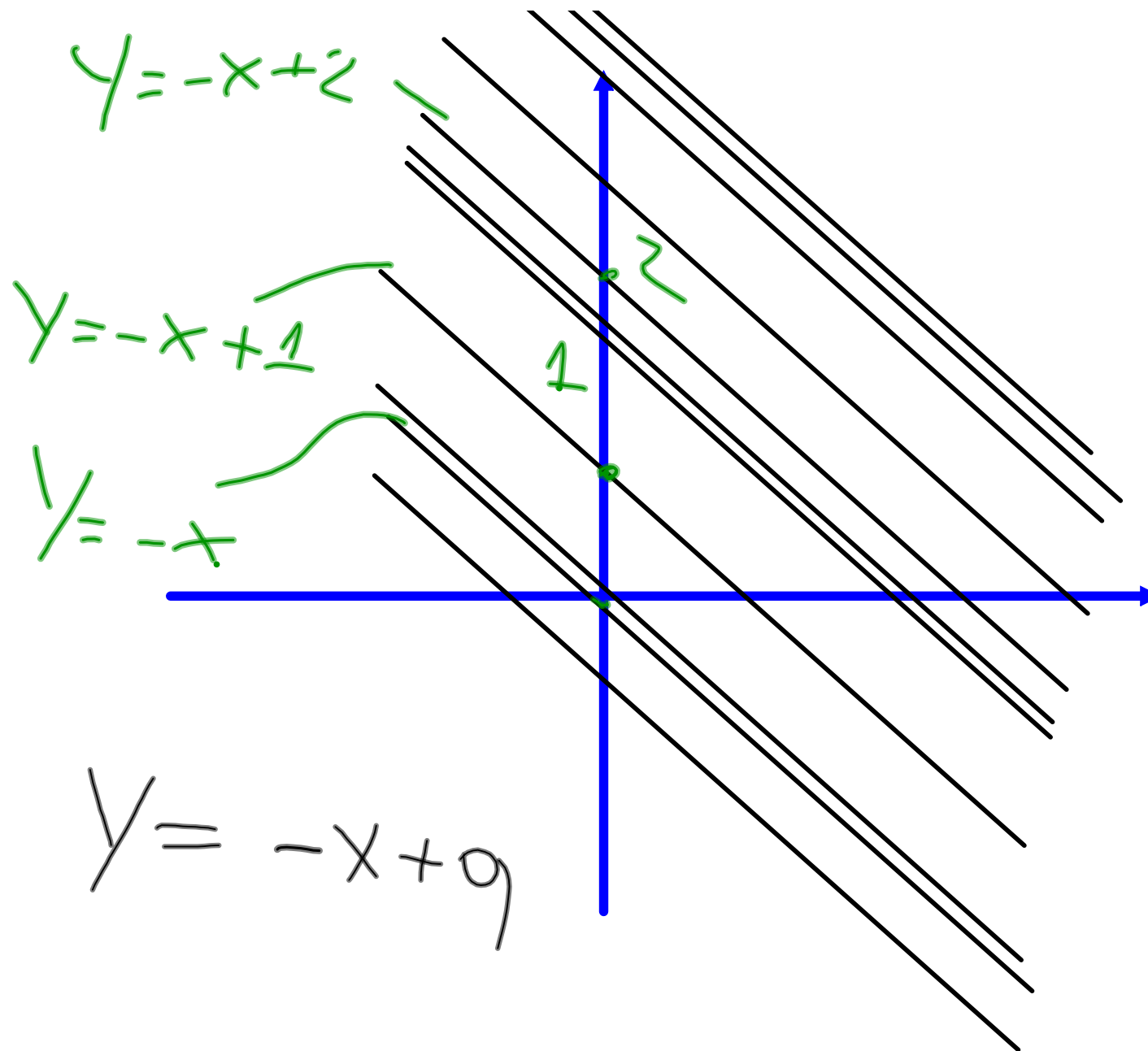


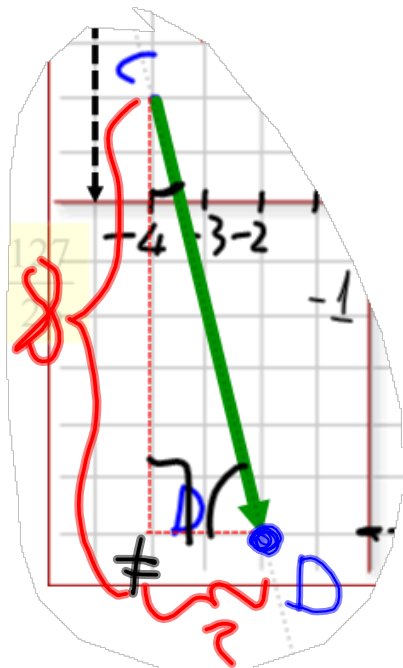
$A(x_A; y_A)$
 $B(x_B; y_B)$

$|x_B - x_A|$

$\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_A - y_B)^2}$

$|y_A - y_B|$





$$\frac{CD}{\sin 90} = \frac{DF}{\sin \hat{C}}$$

$$\sqrt{68}$$

$$= \frac{2}{\sin \hat{C}} \cdot \frac{\sqrt{68}}{\sqrt{68}}$$

$$\sin C = \frac{2}{\sqrt{68}} \cdot \frac{\sqrt{68}}{\sqrt{68}}$$

$$= 14^\circ$$

$$AB = \sqrt{53}$$

$$CP = \sqrt{68}$$

$$\textcircled{I} \quad \frac{-2x + 2y = 7}{2}$$

$$\frac{x - x_D}{x_A - x_D} = \frac{y - y_D}{y_A - y_D}$$

A(-3;5)
B(4;3)
C(-4;2)
D(-2;-6)
O(2;-1)

$$\frac{x+2}{-4+2} = \frac{y+6}{2+6}$$

$$\frac{x+2}{-2} = \frac{y+6}{8}$$

$$\frac{-4(x+2) - y - 6}{8} = 0 \rightarrow -4x - 8 - y - 6 = 0$$

$$-4x - 14 = y$$

$$\textcircled{II} \quad y = -4x - 14$$

$$y = -4x - 14$$

$$y = \frac{-2x + 29}{7}$$

$$-4x - 14 = \frac{-2x + 29}{7} \quad \left. \vphantom{-4x - 14 = \frac{-2x + 29}{7}} \right\} x = \dots = -\frac{127}{26}$$

$$y = -4 \left(-\frac{127}{26} \right) - 14$$

$$\textcircled{1} \quad \frac{-2x + 29}{7} = y$$

$$\textcircled{1} \quad y = -4x - 14$$

$$\begin{cases} y = -\frac{2}{7}x + \frac{29}{7} \\ -y = +4x + 14 \end{cases} \quad (-1) \quad \frac{72}{13}$$

$$y = -4\left(-\frac{127}{26}\right) - 14$$

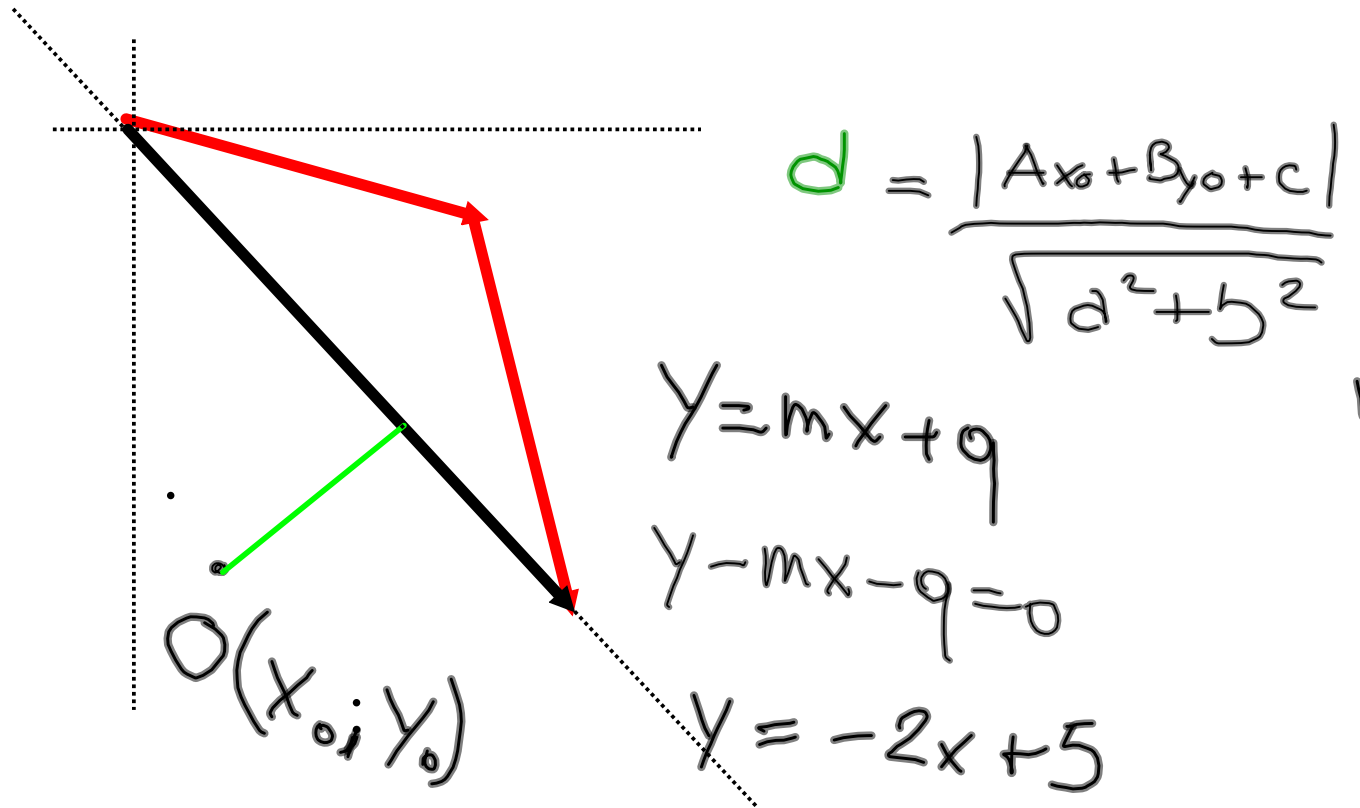
$$y = -2\left(-\frac{127}{13}\right) - 14$$

$$= \frac{254}{13} - \frac{182}{13}$$

$$0 = \left(4 - \frac{2}{7}\right)x + \frac{29}{7} + 14 \Rightarrow x = \dots$$

$$0 = \frac{26}{7}x + \frac{127}{7} \Rightarrow x = -\frac{127}{26}$$

$$\begin{cases} y = 5x + 7 \\ y = -x - 2 \end{cases}$$

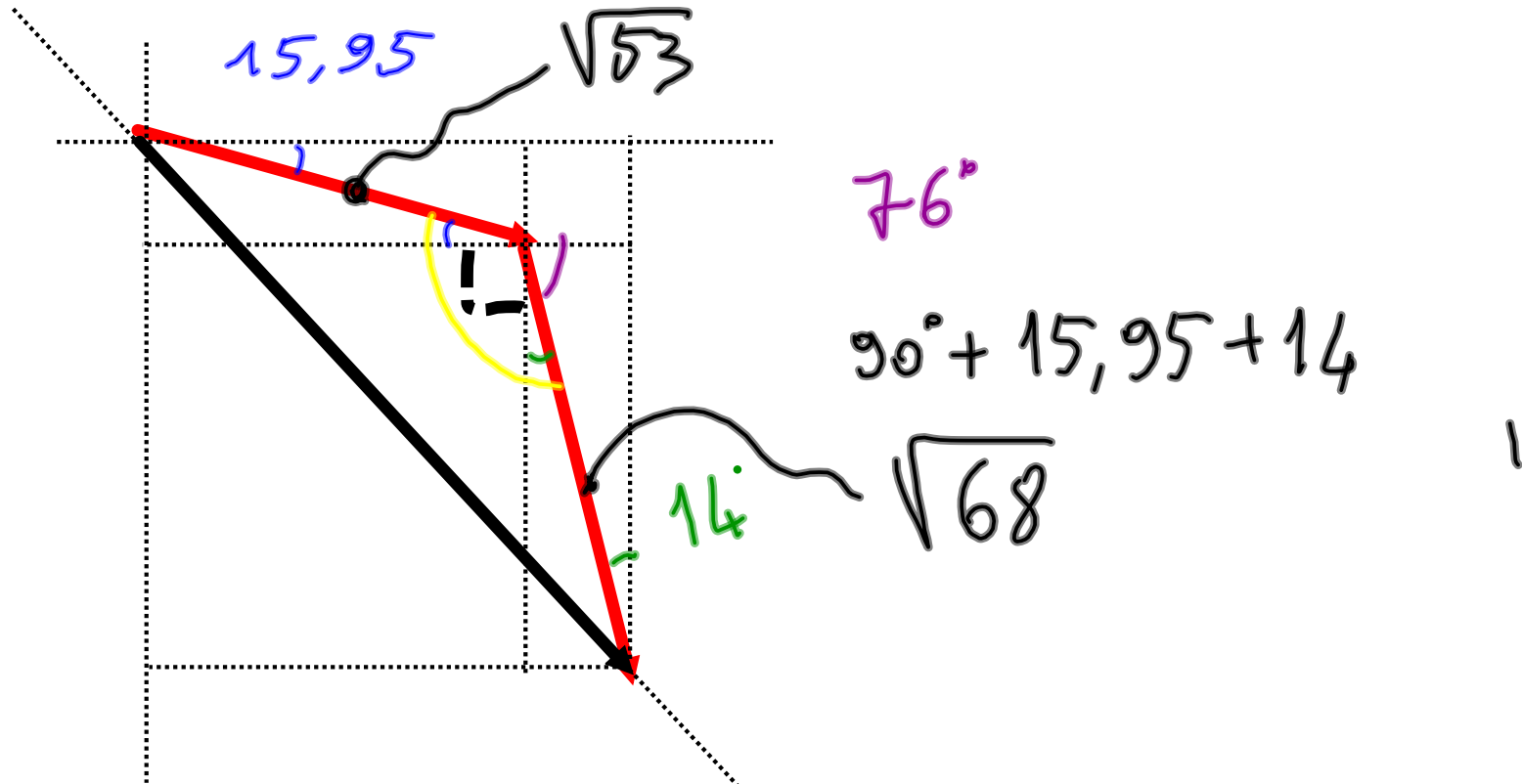


$$2x + y - 5 = 0$$

$$A = 2$$

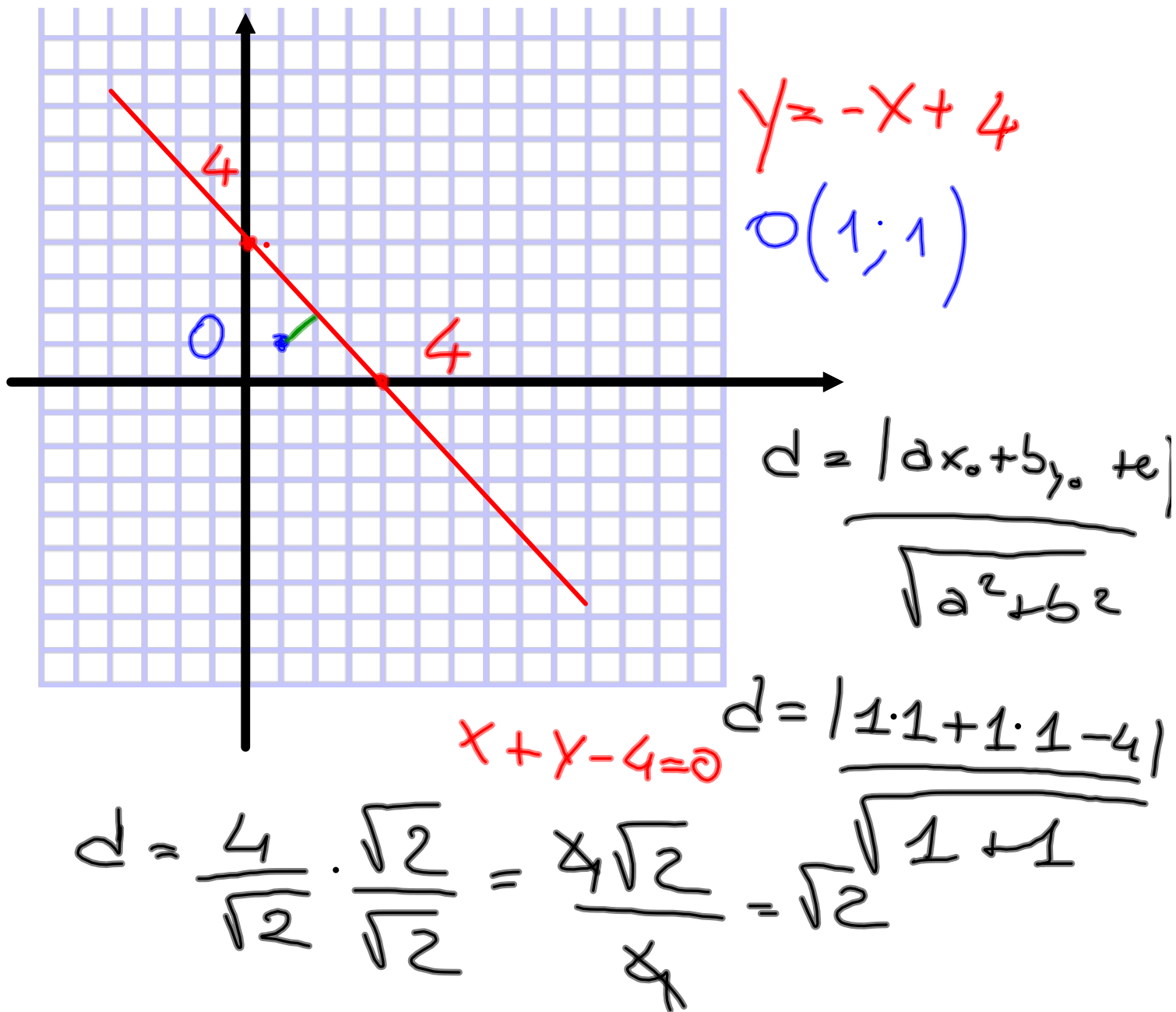
$$B = 1$$

$$C = -5$$



$$F_R = \sqrt{(\sqrt{53})^2 + (\sqrt{68})^2 - 2(\sqrt{53})(\sqrt{68}) \cdot \cos(119^\circ)}$$

$$F_R = 13,45$$



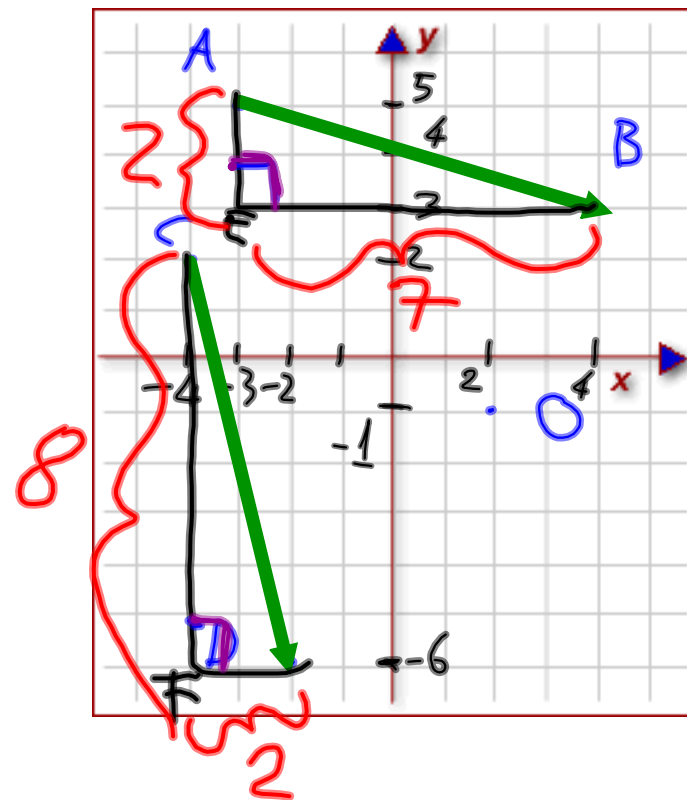
$$AB = \sqrt{4 + 49}$$

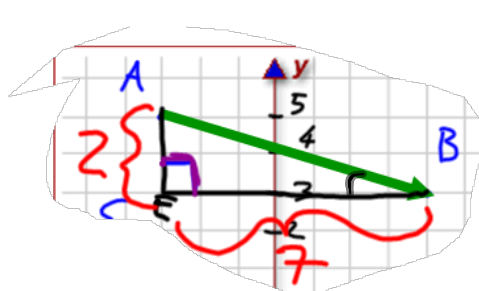
$$= \sqrt{53}$$

$$CD = \sqrt{4 + 64}$$

$$= \sqrt{68}$$

A(-3;5)
 B(4;3)
 C(-4;2)
 D(-2;-6)
 O(2;-1)





$$AB = \sqrt{53}$$

$$CP = \sqrt{68}$$

$$\frac{AB}{\sin \hat{C}} = \frac{AC}{\sin \hat{B}} \rightarrow \frac{\sqrt{53}}{\sin 90} = \frac{2}{\sin \hat{B}} \leftarrow x$$

$$\sqrt{53} = \frac{2}{\sin \hat{B}}$$

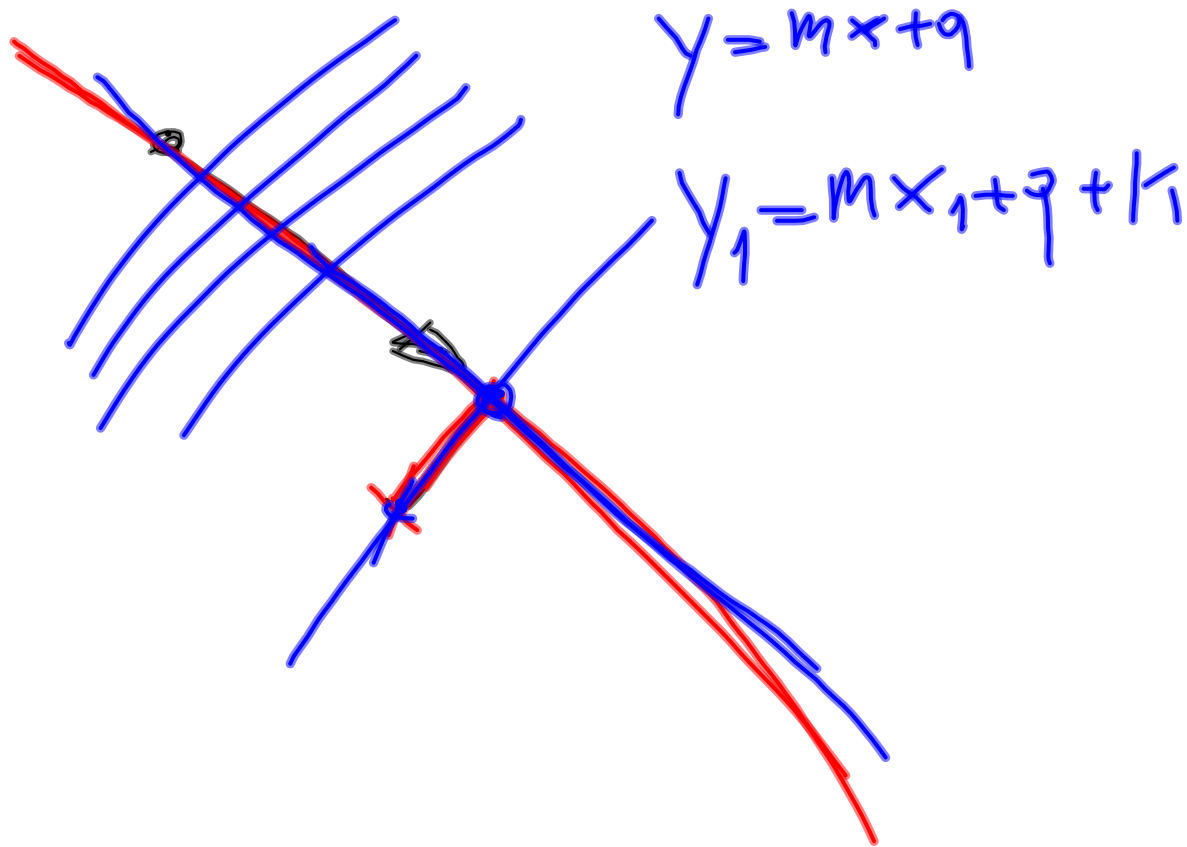
$$\sin \hat{B} = \frac{2}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}}$$

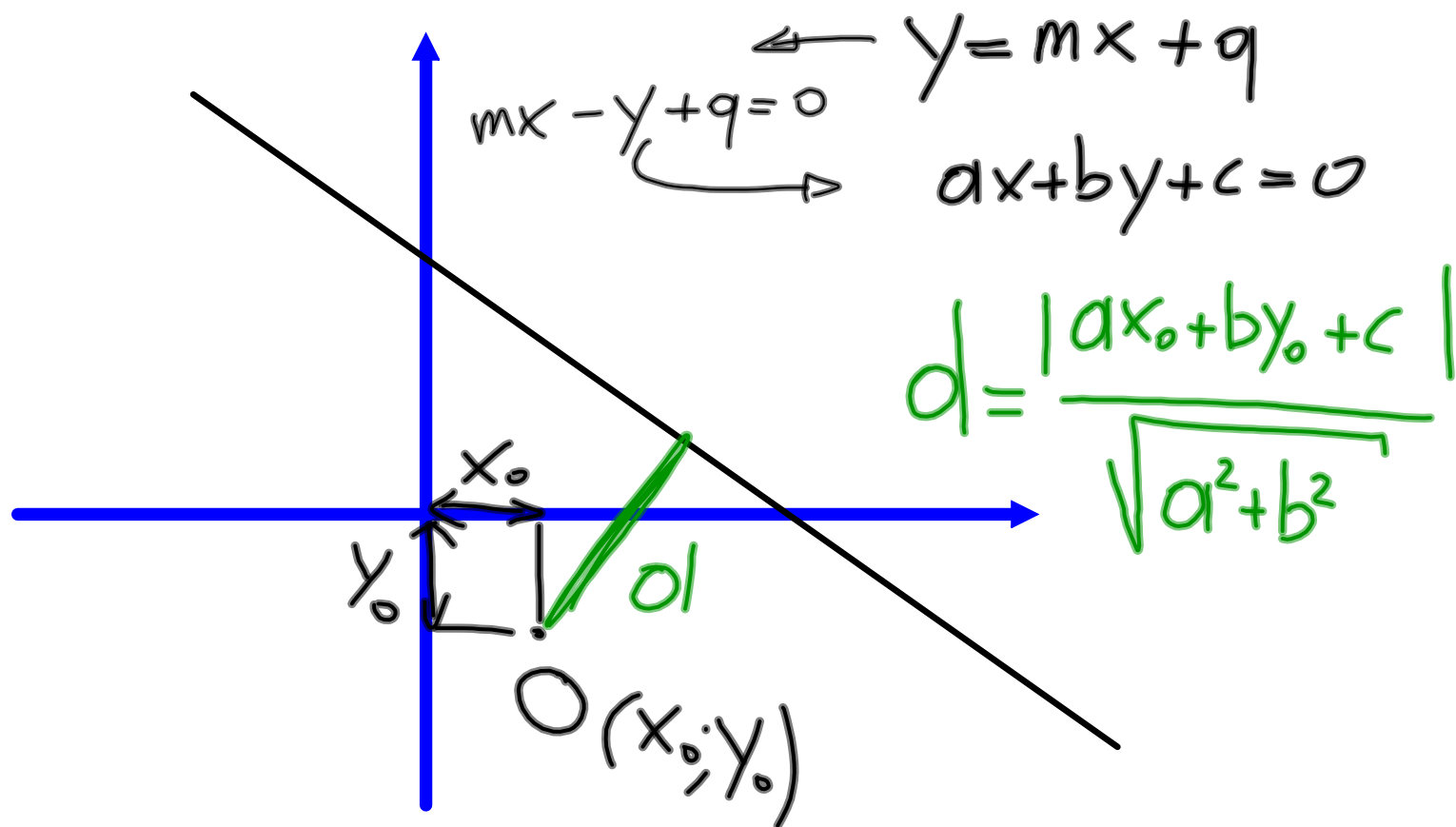
$$\hat{B} = 15,95^\circ$$

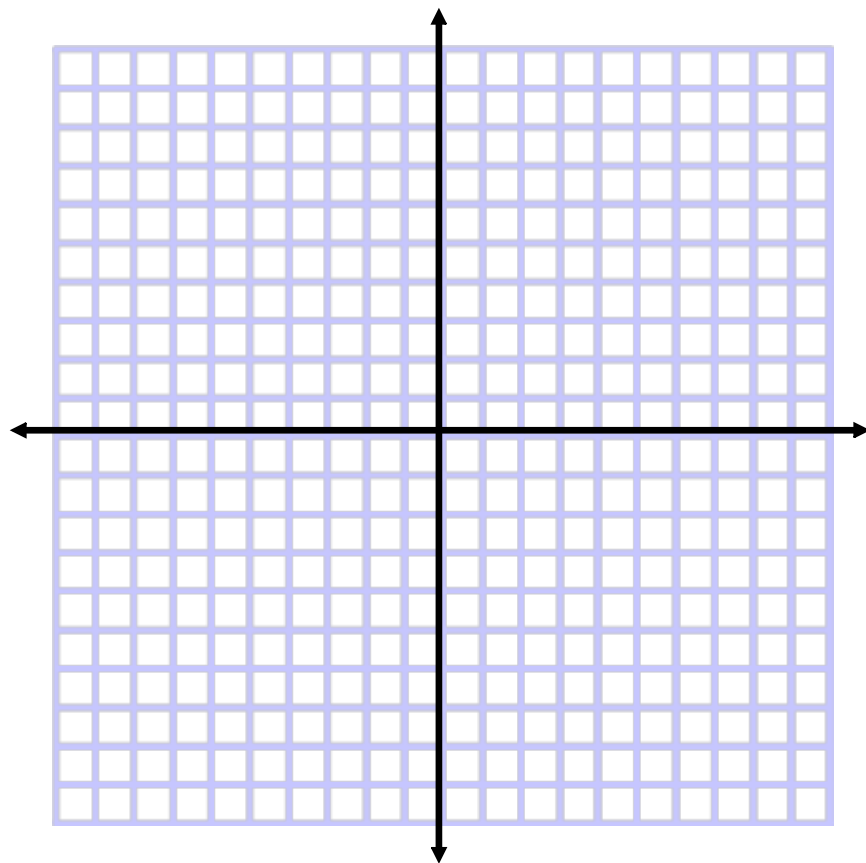
$$\hat{A} = 180^\circ - (90 + 15,95)$$

$$= 180^\circ - 105,95$$

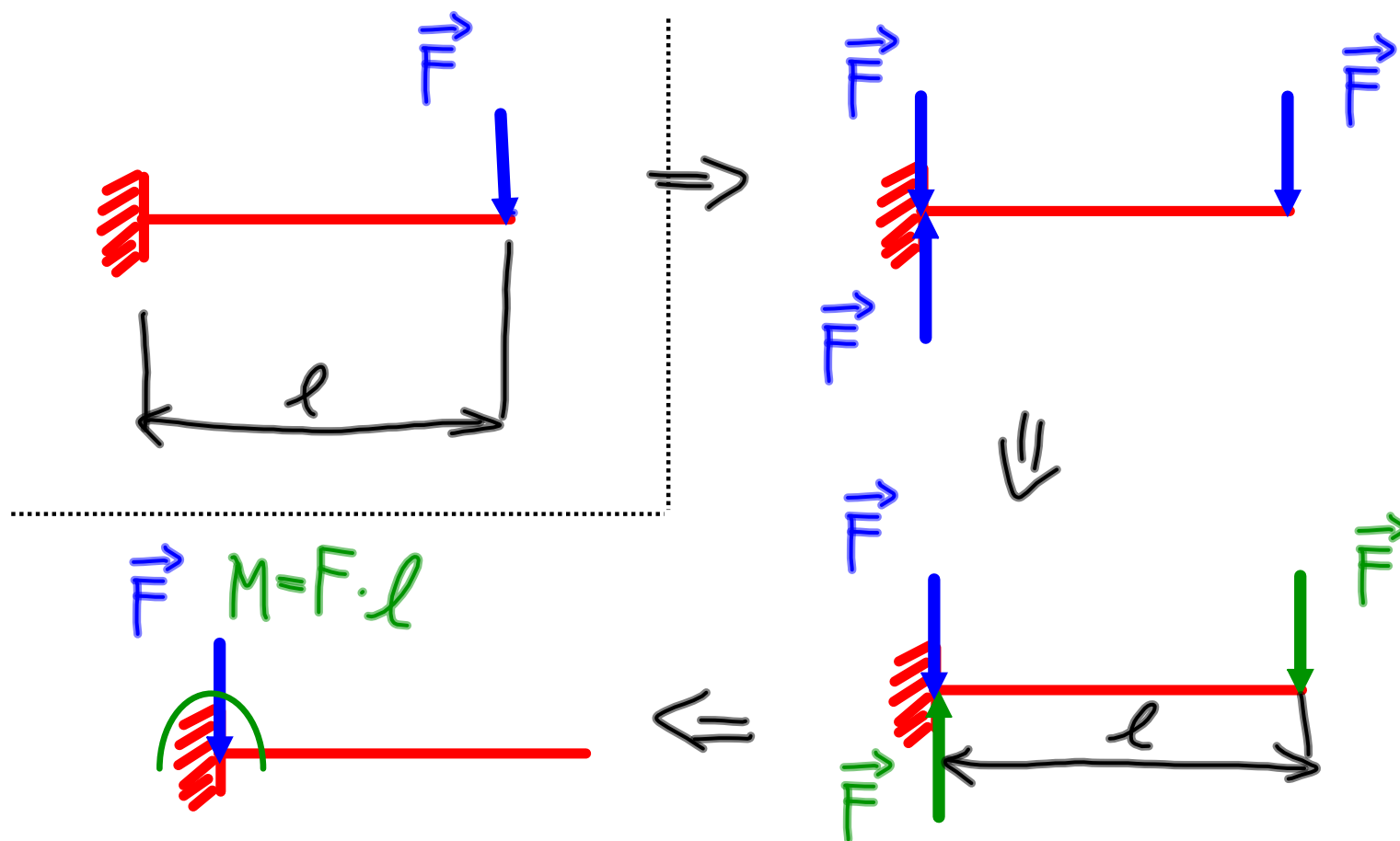
$$\approx 74,05^\circ$$







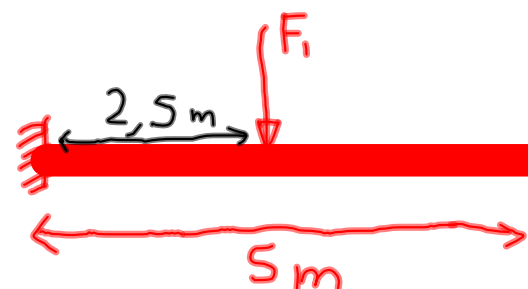
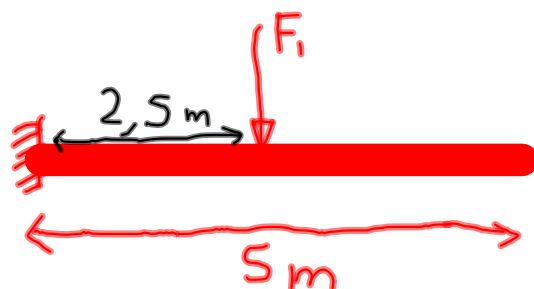
TRASPOSIZIONE DI UNA FORZA



esercizio

→ META'

una forza di 1000N è applicata (verso il basso) in ~~mezzera~~ di una trave lunga 5m. Calcolare il sistema forza-coppia equivalente



esercizio

una forza di 1000N è applicata (verso il basso) in mezzeria di una trave lunga 5m. Calcolare il sistema forza-coppia equivalente

