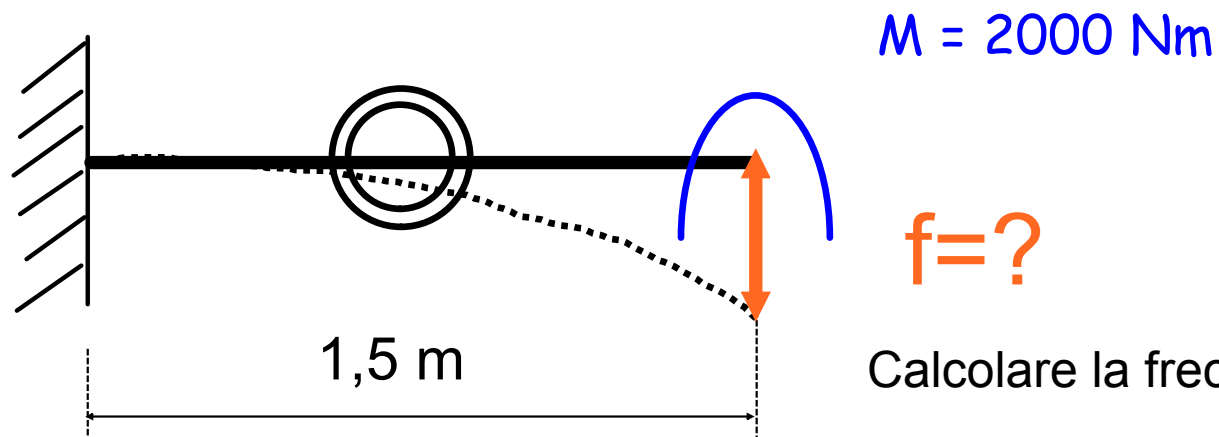


*esercizi*  
di calcolo  
di  
*freccie elastiche*

# ESERCIZIO 1



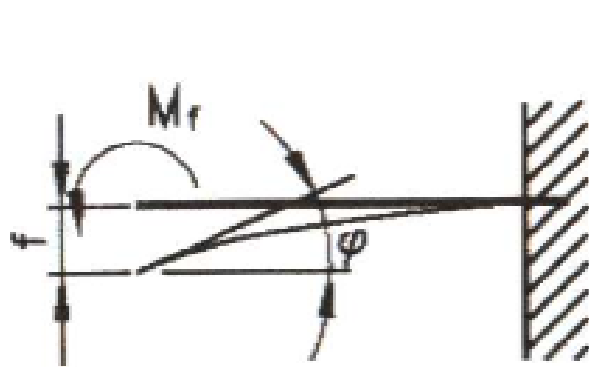
Calcolare la freccia massima ?



N.B.

$E$  (modulo di elasticità / modulo di Young) =  $206000 \text{ N/mm}^2$

# SOLUZIONE 1



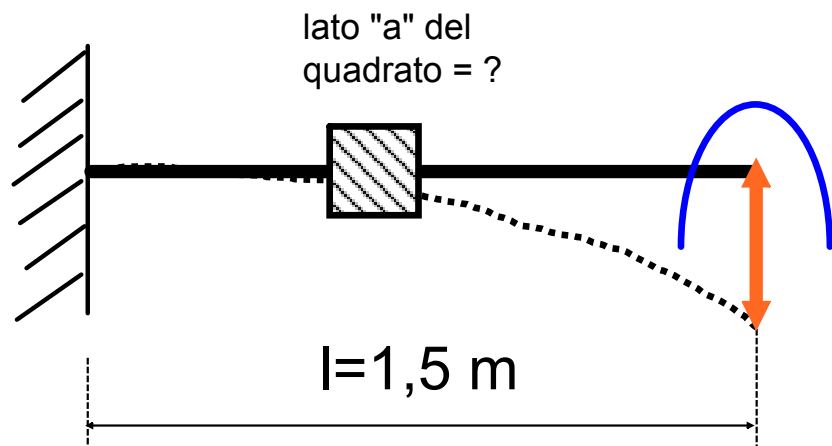
$$f = \frac{M_f \cdot l^2}{2E \cdot I}$$

$$f = \frac{M \cdot l^2}{2 \cdot E \cdot I}$$

$$I = \frac{\pi}{64} (d_e^4 - d_i^4) = \frac{\pi}{64} (60^4 - 50^4) \cong 329376 \text{ mm}^4$$

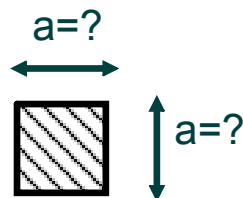
$$f = \frac{2000 \text{ Nm} \cdot (1,5 \text{ m})^2}{2 \cdot 206000 \frac{\text{N}}{\text{mm}^2} \cdot 329376 \text{ mm}^4} = \frac{2000 \cdot 10^3 \text{ Nmm} \cdot (1,5 \cdot 10^3 \text{ mm})^2}{2 \cdot 206000 \frac{\text{N}}{\text{mm}^2} \cdot 329376 \text{ mm}^4} = 33,16 \text{ mm}$$

## ESERCIZIO 2



N.B.  
E (modulo di elasticità / modulo di Young) =  $206000 \text{ N/mm}^2$

Calcolare il lato del quadrato per avere una freccia limite uguale ad  $1/800$  della lunghezza della trave.



## SOLUZIONE 2

$$f_{LIM} = \frac{1}{800} \cdot l = \frac{1}{800} \cdot 1500mm = 1,875mm$$

$$1,875 = \frac{M \cdot l^2}{2 \cdot E \cdot I}$$

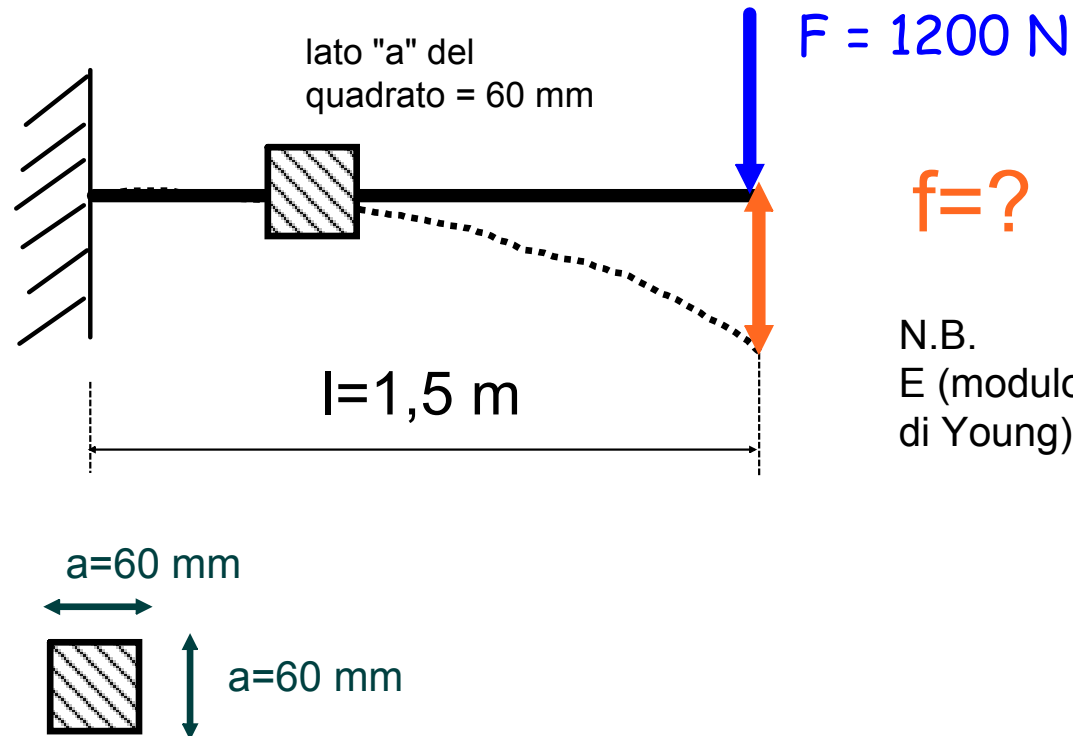
$$I = \frac{M \cdot l^2}{2 \cdot E \cdot 1,875}$$

$$I = \frac{2000Nm \cdot (1,5m)^2}{2 \cdot 206000 \frac{N}{mm^2} \cdot 1,875mm} = \frac{2000 \cdot 10^3 Nmm \cdot (1,5 \cdot 10^3 mm)^2}{2 \cdot 206000 \frac{N}{mm^2} \cdot 1,875mm} = 5825243 mm^4$$

$$\begin{cases} I = \frac{1}{12} a^4 \\ I = 5825243 mm^4 \end{cases}$$

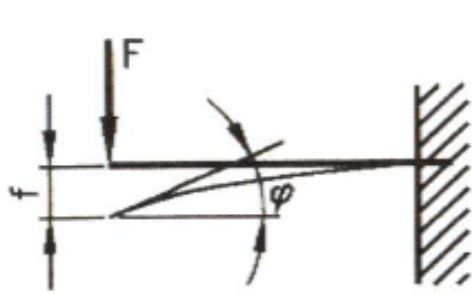
$$5825243 mm^4 = \frac{1}{12} a^4 \rightarrow a = \sqrt[4]{12 \cdot 5825243 mm^4} \cong 91,43mm$$

## ESERCIZIO 3



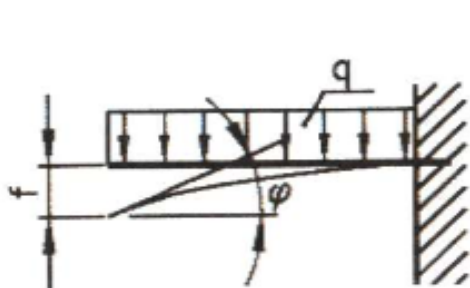
Calcolare la freccia massima considerando sia la freccia dovuta alla forza "F" sia la freccia dovuta alla forza peso "q" della trave ?

## SOLUZIONE 3



$$f = \frac{F \cdot l^3}{3E \cdot I}$$

$$\longrightarrow \mathbf{f_F}$$

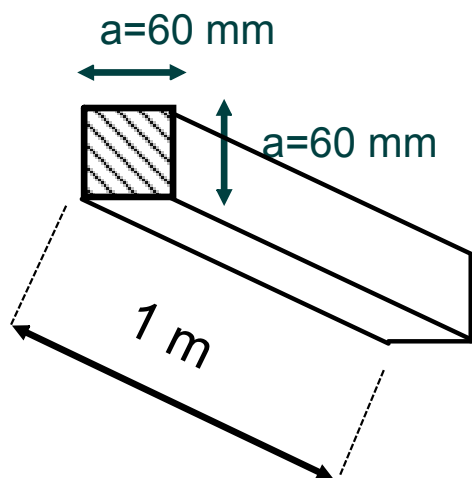


$$f = \frac{q \cdot l^4}{8E \cdot I}$$

$$\longrightarrow \mathbf{f_q}$$

$$\mathbf{f_{totale} = f_F + f_q}$$

$$q = \text{????}$$



$$\rho_{\text{ACCIAIO}} = 7,85 \frac{\text{kg}}{\text{dm}^3}$$

$$m = \rho \cdot V$$

$$m = 7,85 \frac{\text{kg}}{\text{dm}^3} \cdot \text{Sezione} \cdot \text{lunghezza}$$

$$m = 7,85 \frac{\text{kg}}{\text{dm}^3} \cdot \text{Sezione} \cdot 10\text{dm} = 7,85 \frac{\text{kg}}{\text{dm}^3} \cdot (0,6^2 \text{dm}^2) \cdot 10\text{dm} = 28,26\text{kg}$$

$$1\text{m di trave pesa } 28,26 \text{ kg} \rightarrow q = 28,26 \frac{\text{kg}}{\text{m}} \rightarrow 28,26 \cdot 9,81 \frac{\text{N}}{\text{m}} \cong 277 \frac{\text{N}}{\text{m}}$$

$$q = 277 \text{ N/m}$$



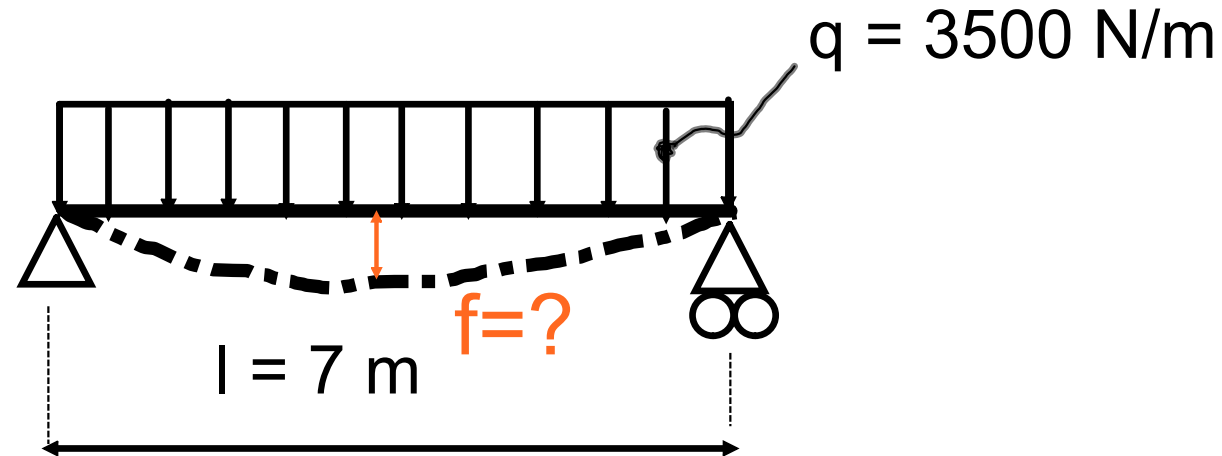
$$I = \frac{1}{12} a^4 = \frac{1}{12} 60^4 = 1080000 \text{mm}^4$$

$$f_F = \frac{F \cdot l^3}{3 \cdot E \cdot I} = \frac{1200 \text{N} \cdot (1500 \text{mm})^3}{3 \cdot 206000 \frac{\text{N}}{\text{mm}^2} \cdot 1080000 \text{mm}^4} \cong 6,07 \text{mm}$$

$$f_q = \frac{q \cdot l^4}{8 \cdot E \cdot I} = \frac{0,277 \frac{\text{N}}{\text{mm}} \cdot (1500 \text{mm})^4}{8 \cdot 206000 \frac{\text{N}}{\text{mm}^2} \cdot 1080000 \text{mm}^4} \cong 0,79 \text{mm}$$

$$f_{\text{totale}} = f_F + f_q = 6,07 + 0,79 = 6,86 \text{ mm}$$

## ESERCIZIO 4



$$I_{\text{trave}} = 869 \text{ cm}^4$$

Calcolare la freccia massima considerando la freccia dovuta alla forza peso "q" della trave ?

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## SOLUZIONE 4

$$f = 61,1 \text{ mm}$$